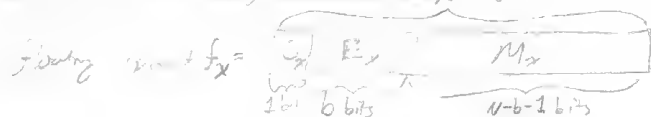


Fast Inverse Square Root

(0x5f3759df)



is only defined for $x \neq 0$

$$\therefore f_x = (1 + \frac{M_x}{2^{N-b-1}}) 2^{E_x - (2^{b-1} - 1)}$$

For simplicity, let $L = 2^{N-b-1}$, which represents the mantissa M to $0 \leq M < 1$

& let $B = 2^{b-1} - 1$, which is the exponent bias.

$$f_x = (1 + \frac{M_x}{L}) 2^{E_x - B} \quad (1)$$

Looking for $y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Let f_x & f_y be floating point representations of x & y respectively.

$$\therefore f_y = f_x^{-\frac{1}{2}} \quad \text{ignoring errors introduced by floating point}$$

$$\log_2 f_y = \log_2 (f_x^{-\frac{1}{2}})$$

$$\log_2 f_y = -\frac{1}{2} \log_2 f_x$$

$$\log_2 \left((1 + \frac{M_x}{L}) 2^{E_x - B} \right) = -\frac{1}{2} \log_2 \left((1 + \frac{M_x}{L}) 2^{E_x - B} \right)$$

$$\log_2 (1 + \frac{M_x}{L}) + \log_2 (2^{E_x - B}) = -\frac{1}{2} \left[\log_2 (1 + \frac{M_x}{L}) + \log_2 (2^{E_x - B}) \right]$$

$$\log_2 (1 + \frac{M_x}{L}) + E_y - B = -\frac{1}{2} \log_2 (1 + \frac{M_x}{L}) - \frac{1}{2} E_x + \frac{1}{2} B$$

$$\log_a(1 + \frac{1}{2} \epsilon^2) + E_y = \frac{1}{2} \log_a \frac{1}{2} + \frac{1}{2} \log_a \frac{1}{2} - \frac{1}{2} E_x + \frac{1}{2} \log_a \frac{1}{2}$$

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substituting (3) into (2) we have:

$$\frac{M_y}{L} + E_y = \frac{1}{2} \log_a \frac{1}{2} + \frac{1}{2} \log_a \frac{1}{2} - \frac{1}{2} E_x + \frac{1}{2} \log_a \frac{1}{2}$$

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$$E_y L + M_y = \frac{1}{2} \log_a \frac{1}{2} + \frac{1}{2} \log_a \frac{1}{2} - \frac{1}{2} E_x L + \frac{1}{2} \log_a \frac{1}{2}$$

$$= \frac{1}{2} \log_a \frac{1}{2} + \frac{1}{2} \log_a \frac{1}{2} - \frac{1}{2} E_x L + \frac{1}{2} \log_a \frac{1}{2}$$

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Now we can look at the first term

N=1

$$2^{N-1} M = \frac{1}{2} \log_a \frac{1}{2} + \frac{1}{2} \log_a \frac{1}{2} - \frac{1}{2} E_x L + \frac{1}{2} \log_a \frac{1}{2}$$

if we take I_x and T_0 as the I_x and T_0 for I_x and T_0 , respectively, for the integers, then we have:

$$I_x = E_x L + M_x, \quad I_y = E_y L + M_y.$$

So from (5), (6) and (7) we have:

$$I_y = L \left(\frac{3}{2} B - \left(\frac{1}{2} \theta_x + \theta_y \right) \right) - \frac{1}{2} I_x$$

or more simply:

$$I_y = R - \frac{1}{2} I_x$$

$$\text{where } R = L \left(\frac{3}{2} B - \left(\frac{1}{2} \theta_x + \theta_y \right) \right)$$

And that's the basic technique: take floating point I_x and I_y and subtract one magic integer R from I_y . If you take I_x as a floating point you get I_y .

The error comes from the fact that I_x and I_y are specific values of x (i.e. $x = 1$).

So we need to pick I_x and I_y as best as we can as an approximation for most values of x .

For example, if we take $I_x = 1$ and $I_y = 1$, then

$$I_y = 1$$

$$I_y = 1 \left(\frac{3}{2} (1.27) - \left(\frac{1}{2} \theta_x + \theta_y \right) \right)$$